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Reliability modeling for multi-component system considering self-healing and dependent competing failure process

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Abstract

In the evaluation of system reliability based on dependent competing failure process, it is essential to consider the effect of self-healing, and ignoring self-healing may lead to errors in the evaluation results. Existing models only considers the self-healing in soft failure or hard failure, without considering self-healing of both failure processes simultaneously which lacks reality. In this paper, a general reliability model for multi-component system based on different shock models is established. During the soft failure process, a first-kind self-healing factor is introduced to make the degradation increment change with time in this model. Two second-type self-healing factors are introduced into the hard failure process, which causes the hard failure threshold under extreme shock model and the cumulative damage of cumulative shock model to change with the shock magnitude and numbers of shock. A new reliability model for a parallel-series system is established and the analytical expression is derived through combining dynamic self-healing mechanism with multi-component system. Finally, an illustrative example of self-healing gearbox is given to verify the feasibility of the model.

Keywords: self-healing, multi-component, dependent competing failure process, random shock, internal degradation

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Notation

H_i	Soft failure threshold of the ith component
$D_i(t)$	Hard failure threshold of the ith component
$S_i(t)$, $X_i(t)$	Continuous degradation and degradation increment of the ith component
$X_{s,i}(t)$	Overall degradation of the ith component
β_i	Degradation rate of the ith component
W_{ik}	The magnitude of the kth shock on the ith component
Y_{ik}	The degradation increment of the kth shock on the ith component
$N(t)$	Number of shocks arrived at time t
λ	Arrival rate of the Homogeneous Poisson Process
γ_i	Soft process self-healing factor
ς_1, ς_2	Hard process self-healing factor
$R_{i,\mathrm{SF}}$	Soft failure reliability of the ith component
$R_{i,\mathrm{SH}}$	Hard failure reliability of the ith component
$R_{i,e}$, $R_{i,c}$	Reliability of the ith component subjected to extreme shock model and cumulative shock model
$R_{i,e1}$, $R_{i,e2}$	Reliability of the ith component subjected to extreme shock model when no shocks and existing shocks
$R_{i,c1}$, $R_{i,c2}$	Reliability of the ith component subjected to cumulative shock model when no shocks and existing shocks
$R_{s,e}$, $R_{s,c}$	Reliability of the multi-component system subjected to extreme shock model and cumulative shock model
c_1	Transfer parameters of degradation increment
c_2, c_3	Transfer parameters of payload

1. Introduction

The effect of self-healing has gained increasing attention within the field of reliability analysis in recent years. Considering self-healing makes the established model more in line with the actual engineering situation. Many engineering systems possess self-healing capability due to the self-recovery materials used in manufacturing processes, including the self-healing materials of polymer matrix composites products [1] and self-repairing coating materials applied to pipelines [2, 3]. Therefore, it is crucial to consider self-healing effect in reliability evaluation of the system.

Components with different performances collectively constitute a system, and any component failure may cause the failure of system. There has been more significant attention given to the analysis and development of reliability models based on failure mechanisms in recent years [4–6]. The external random shocks and internal progressive degradation are considered the two primary factors related to the system failure [7–9]. Generally, the internal degradation process is characterized by gradual decline in performance of products caused by wear, fatigue, and corrosion, which lead to soft failure of system ultimately. Meanwhile, when the magnitude of external shocks exceeds critical threshold, the system may suddenly stop working and fail catastrophically. The processes that result in system failure are competing and interdependent. The system model used dependent competing failure processes (DCFPs) provides a more detailed understanding of failure modes and offers an indirect approach for estimating system reliability.

Random processes have been shown to describe both internal deterioration process and external impact process of products effectively. For the internal degradation that leads to soft failure of system, general degradation path models and stochastic degradation models have been introduced for analyzing system reliability during the soft failure process (SFP) [9–11]. The important issue of degradation modeling is to seek the correlation in both the degradation path and the life function of system. As to hard failure process (HFP) of system, it is crucial to select a proper random shock model. Several kinds of hard failure models have been identified, including the extreme shock model [12, 13], δ-shock model [14], cumulative shock model [15, 16], m-shock model, and run shock model [17]. Different shock models should be selected appropriately based on varying external conditions to develop the reliability models more effectively.

For the research of self-healing materials, Kessler [18] and Davis *et al* [19] developed a kind of self-healing material, which is widely applied in polymer-based composite products. For instance, polymer adhesives are used to mitigate volume changes in silicon in micro-electro-mechanical-systems (MEMSs). The polymer network structure and self-healing properties within the electrodes are able to buffer the strain caused by volume change of silicon [4]. Yang *et al* [20] proposed a multi-parameter ultrasonic approach to evaluate and predict strength recovery in self-healing cement-based materials by linking ultrasonic features to the evolution of healing-induced mechanical restoration. Focusing on systems with self-healing capabilities, many researchers have conducted different studies. Zhao *et al* [21] and Kang and Cui [22] investigated the influence of self-recovery impact for system reliability under a two-stage mixed shock model and two different shock models, respectively. Li *et al* [23] correlated the self-healing with shock load and environmental effects, and analyzed the reliability of the developed system. Qiao *et al* [24] developed a reliability model for products with self-healing under dynamic shock environments and applied it to warranty optimization by characterizing the coupled effects of shock loading and self-healing behavior on product reliability.

In recent years, the reliability of multi-component system experiencing DCFP has also gradually become a focal point. Lyu *et al* [25] analyzed DCFPs by introducing a time-varying δ-shock model to characterize the influence of nonstationary shock effects on system reliability. Sun *et al* [26] established a reliability model using a nonlinear Wiener degradation process and the copula approach with time-varying, demonstrating the practicality through a MEMS case study. Lyu *et al* [27] incorporated the failure propagation effect into the DCFP and introduced discrete failure time to analyze the reliability of multi-component systems. Liang *et al* [28] investigated the reliability of multi-component systems operating in dynamic environments by modeling the coupled effects of dependent degradation processes and random shocks on system reliability. Lyu *et al* [29] studied the reliability of the system with two components and three shock sources, in which two shock sources affect their respective components, while one shock source affects both components simultaneously. Zhang *et al* [30] studied the dynamic opportunistic maintenance of the multi-component system for load sharing through deep reinforcement learning.

However, the research on multi-component system considering self-healing capability remains relatively limited in the existing literature. Wang *et al* [31] developed a whole-life-cycle reliability model for products with self-recovery features by modeling competing failure processes and recovery effects within a stochastic reliability framework. Yang *et al* [32] analyzed a partially repaired self-healing multiple DCFPs (MDCFP) system. Cui *et al* [33] introduced the concept of self-healing effect on system and established a new damage cumulative shock model, which considered different counting processes. Liu *et al* [34] analyzed the reliability of system related to MDCFP with self-healing, in which the self-healing process is described by the recovery time and level. However, the reliability function was not obtained in an analytical form due to the complex self-healing process. Shen *et al* [35] investigated the self-healing mechanism of the system under external shocks, and obtained different performance indexes of system. Furthermore, Wang *et al* [36] analyzed the reliability of self-healing systems with multi-component protective devices operating in shock environments by modeling the effects of random shocks and self-healing recovery on system performance. Qi *et al* [37] studied the humidity sensors with self-healing mechanism and derived a general reliability expression. But only the reprocessing level related to shock intensity was considered in their study. Actually, most studies simplify the processes of self-healing because of the complexity of reliability derivation, which may cause the evaluation results of system reliability inaccurately.

Most existing research focuses on combining self-healing capabilities with shock models, or considering the effect of self-healing on SFP. However, when the system experiences competition related failures, it is necessary to consider the joint effect of different self-healing processes on the system. Neglecting the analysis of the shock and degradation processes simultaneously affected by the self-healing may lead to inaccurate reliability assessment. Although self-healing mechanisms have been incorporated into reliability modeling in recent years, most studies consider self-healing only in either SFP or HFP. The limitation is mainly due to several challenges. Firstly, the SFP and HFP differ in the temporal characteristics and mathematical descriptions. The SFP is modeled as a continuous degradation process, while the HFP is triggered by discrete random shocks. Introducing self-healing into both the SFP and HFP simultaneously requires a framework that addressing continuous degradation recovery as well as damage and threshold recovery, which increases modeling complexity. Secondly, the physical self-healing mechanisms of the two failure modes are different. The distinct mechanisms require different functional representations, which increases the difficulty of reliability modeling. In addition, the self-healing effects occurring in the SFP and HFP are often difficult to distinguish. The observed self-healing phenomenon may result from reduced degradation, threshold restoration, or the combination of both effects. Consequently, most existing studies simplify the modeling by incorporating self-healing into only a single failure process. In practical mechanical systems such as gearboxes, bearings, and transmission systems, self-healing technologies have been introduced to mitigate both gradual degradation (e.g., wear and fatigue) and sudden shock-induced failures (e.g., tooth breakage). These systems provide a realistic engineering background for jointly considering self-healing effects in SFP and HFP and prompt us to establish a new reliability model combined with multi-component system affect by self-healing based on DCFP.

Through the comprehensive analysis of previous researches, the following research gaps are identified. Firstly, the current research on self-healing mainly focuses on the shock magnitude and environmental impact of HFP with self-healing, or the deterioration increment of SFP with self-recovery capability. However, the reliability issue of industrial systems where HFP and SFP are simultaneously affected by self-healing has not been studied yet. Secondly, previous studies have relatively lacked characterization of self-healing modes for hard failures. Most researchers have extensively studied HFP with self-healing ability based on the cumulative shock model. But in practical engineering, due to the diversification of external shocks to the system, the self-healing performance mode of the system varies under different shock models. Therefore, we establish a new self-healing model to fill the research gap.

A reliability model for multi-component system based on DCFP, incorporating self-healing in both SFP and HFP, is established in this study. The model considers self-healing factors associated with shock intensity and numbers of shock, which indirectly represent the self-healing recovery level. In addition, the detailed self-healing function is established to reflect the impact of system with self-healing. Finally, the reliability expression of system under different models is derived. A self-healing gearbox is used as a case study to validate the reliability model. The main contributions of this paper are listed as follows:

(1) A unified DCFP model is developed, in which self-healing mechanisms are simultaneously incorporated into both the SFP and HFP. Unlike existing studies that consider the self-healing under only a single failure mode, the proposed framework integrates the combined effects of degradation recovery in SFP and shock-induced damage repair and threshold recovery in HFP on system reliability.
(2) The proposed model subdivides multiple modes of self-healing by establishing the correlation between the cumulative degradation, random shock, failure threshold,

and self-healing mechanism. The phenomenon of self-healing manifests as a certain degree of recovery of hard failure threshold and the reduction in cumulative damage and degradation increments.

(3) The model is further extended to complex multi-component systems, which implements the reliability evaluation of the system with self-healing capability of two failure processes based on DCFP. The general reliability expression for the parallel series system is derived.

(4) A representative self-healing gearbox case study is presented to validate the feasibility of the proposed reliability model. Compared with models that consider self-healing mechanism in a single failure process to verify the rationality of the model and evaluate system reliability and sensitivity effects of various parameters.

The remainder of the paper is organized as follows. Section 2 describes the framework of the model and gives some basic assumptions. Section 3 derives the reliability formula of the system. In section 4, a numerical example is provided to demonstrate the effectiveness of the developed model. Finally, the concluding remarks are given in section 5.

2. System description and model assumption

Some significant assumptions and descriptions of the system reliability model in this study are made as follows:

(1) All components perform from the as-good-as-new state at the initial time. The linear degradation path is used to describe the degradation path at time t for the ith component. This continuous degeneration process can be depicted by $X(t) = \varphi + \beta t + \varepsilon$.

(2) External random shocks will not only elevate the degree of damage on each component, but also deteriorate internal degradation rate of each component. The degradation rate will change with arrival of each random shock.

(3) Assumed that the magnitude of the kth random shock on the ith component is W_{ik}, which follows a normal distribution. It should be emphasized that the shock load in this assumption does not imply that the magnitude of each individual shock can be directly measured.

(4) In practical systems, the self-healing may be partial or still ongoing when the random shock occurs subsequently. The different situations of self-healing are given as follows:

$$H(t) = \begin{cases} h(t) = \begin{cases} \mathrm{e}^{-\gamma t}, t \geq 0, \\ 0, \text{otherwise}; \end{cases} \\ h(t) = \begin{cases} \mathrm{e}^{-\gamma t}, t \in [0, \kappa], \kappa \text{ is a specified positive constant,} \\ 0, \text{otherwise}; \end{cases} \\ h(t) = \begin{cases} \mathrm{e}^{-\gamma t}, t \in \left[0, T_{N(t)+1} - T_{N(t)-k}\right], \\ 0, \text{otherwise.} \end{cases} \end{cases} \tag{1}$$

We assume that the last self-healing behavior can be completed before the arrival of each random shock. Assuming that the self-healing behavior is continuous, based on $h(t) = \mathrm{e}^{-\gamma t}$, the self-healing mechanism is combined with the number of random shock k to construct the self-healing function. The degree of self-healing for each component is different after each shock.

(5) To ensure that all variables following a normal distribution are non-negative, the standard deviation of the variables is much smaller than their mean, that is, $\mu \gg \sigma$.

Figure 1 shows an example with the evolution of the SFP and the HFP of the ith component under two different shock models. In figure 1(a), the component degradation rate changes with the arrival of external impacts. Due to self-healing capability in SFP, the degradation increment caused by each shock will be partially recovered. The HFT of the component will decay under the external shocks. The self-healing capability of the HFP will make a certain degree of recovery in the reduced threshold. Figure 1(b) shows an example with the evolution of the SFP and the HFP of the ith component under cumulative shock model. It is shown from figure 1(b) that due to self-healing capability of the component, the cumulative damage of hard failure declines with the increase of the number of shocks. In the subsequent, soft failure is defined as the total degradation $X_{s,i}(t)$ reaches the soft failure threshold H_i. Hard failure occurs when the load of the extreme shock W_{ik} and the cumulative shock damage $W_i(t)$ exceed the HFT D_i. In addition, soft failure as well as hard failure will result in system failure. The main principle of the developed model is illustrated in figure 2.

3. Reliability model considering DCFP

3.1. Reliability model of SFP

Under complex working conditions, the degradation process of the system is usually nonlinear. Random processes, such as the Gamma process, have the characteristic of monotonic increase and nonlinearity, which can effectively simulate the internal degradation of the system. In order to simplify the system modeling and enable clear analytical into the interaction between degradation, shocks, and self-healing mechanisms, the general linear degradation process is used to describe the internal degradation process in this study.

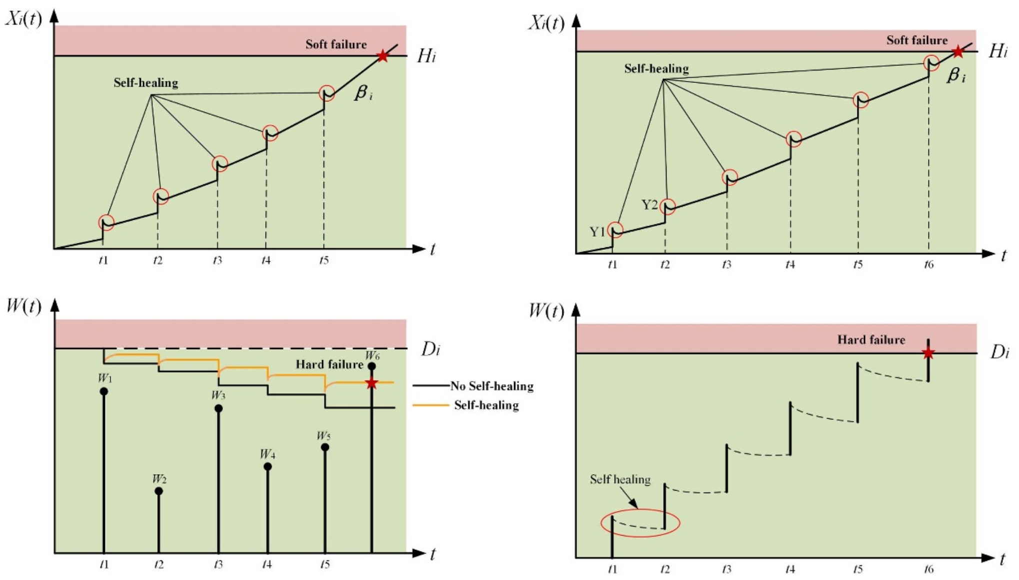
The linear degradation path of the ith component is expressed by equation (2)

$$X_i(t) = \varphi_i + \beta_i t + \varepsilon_i, \tag{2}$$

where ϕ_i and β_i respectively represent the initial values of the natural deterioration and natural deterioration rate of ith component and ε_i is the random error that follows a normal distribution $\varepsilon \sim N\left(0, \sigma^2_{\varepsilon_k}\right)$.

When the ith component is subjected to the jth shock, the degradation rate changes from $\beta_{i,j}$ to $\beta_{i,j+1}$. The degradation rate vector is represented by β

$$\beta = \left[\beta_{i0}, \beta_{i1}, \mathrm{L}, \beta_{ij}, \mathrm{K}\, \beta_{i(n+1)}\right], \tag{3}$$



(a) Extreme shock model (b) Cumulative shock model

Figure 1. Failure process of the ith component with self-healing. (a) Extreme shock model, (b) cumulative shock model.

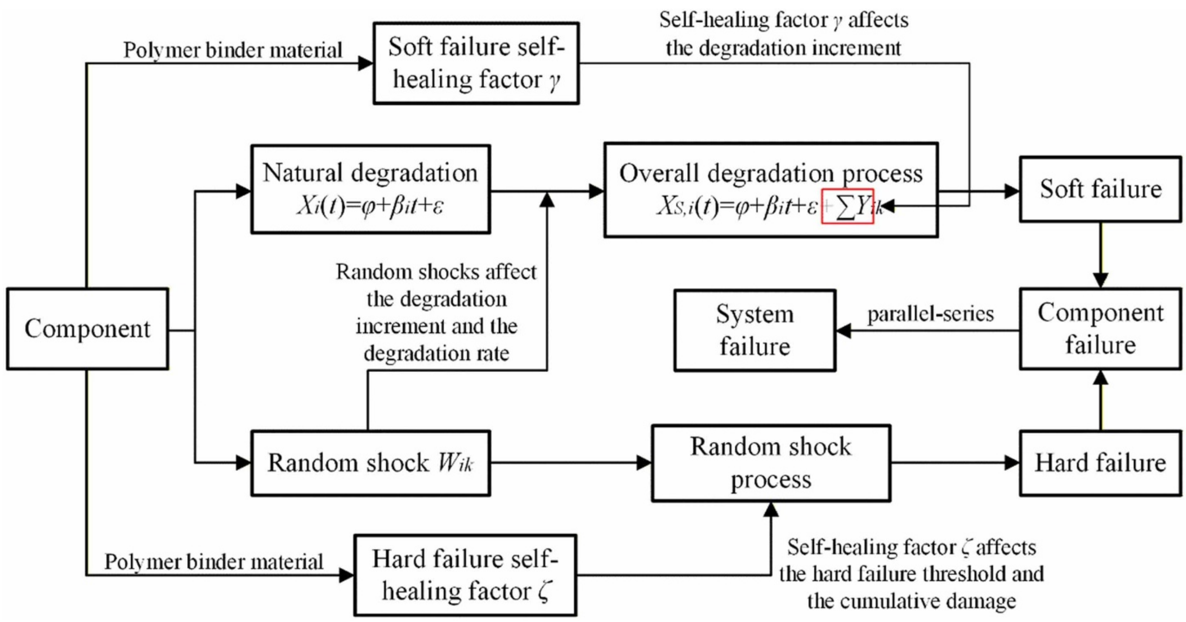


Figure 2. Basic principle of the developed model.

where β_{i0} is the initial value of the natural degradation rate of the ith component, following a normal distribution, $\beta_{i0} \sim N(\mu_{\beta_{i0}}, \sigma^2_{\beta_{i0}})$.

Upon the occurrence of random shocks, the degradation rate of components changes accordingly. It is assumed that the degradation rate increment of the ith component under each random shock is ω_{ij}, where ω_{ij} represents independent and identically distributed (i.i.d.) positive random variables. Furthermore, β_{ij} and ω_{ij} are independent. The relationship is expressed as $\beta_{i,j+1} = \beta_{ij} + \omega_{ij}$.

The time interval between $(j-1)$th and jth shocks is denoted as T_j. We define the vector of time intervals between each shock as T, described as

$$T = [t_1, t_2, \ldots t_j, \ldots, t_n, t_{n+1}]. \tag{4}$$

It is crucial to determine the distribution of the time step T_j for the degradation rate as each shock alters the degradation rate. The arrival time of the shocks, T_j, follows a Gamma distribution with a scale parameter k_j and a shape parameter λ, where $q = 1, 2$ [9]. The pdf is given as follows

$$f_{T_j}(t_j) = \frac{(\lambda p_q)^{k_j}}{(k_j-1)!} t_j^{k_j-1} e^{-\lambda p_q t_j}, \text{for } j = 1,2,\ldots,n.q = 1,2. \tag{5}$$

Assumed that $N(t)$ as the number of shocks experienced to ith component at time t, following a HPP with an arrival rate λ. The magnitude of the kth random shock is denoted as $W_{ik,k=1,\ldots,N(t)}$, which is i.i.d and follows a normal distribution, $W_{ik} \sim N\left(\mu_{w_{ik}}, \sigma^2_{w_{ik}}\right)$.

The random shock process follows the HPP, the probability $P(N(t) = m)$ is expressed based on the arrival rate λ, which is described by

$$P(N(t) = m) = \frac{e^{-\lambda t}(\lambda t)^m}{m!}, m = 0,1,2,\cdots N(t). \tag{6}$$

It is supposed that Y_{ik} (where $k = 1, 2, \ldots, \infty$) represents the degradation increment caused by the kth shock on the ith component. The degradation increment Y_{ik} is related to the shock load W_{ik} at time t. The total degradation increment $S_i(t)$ is described by the following equation (7)

$$S_i(t) = \begin{cases} 0, N(t) = 0 \\ \sum_{k=1}^{N(t)} Y_{ik}, N(t) > 0 \end{cases} \tag{7}$$

The total degradation of the ith component $X_{s,i}(t)$ is the sum of its internal degradation and degradation increments. Therefore, the total degradation of the ith component at time t is obtained by

$$X_{s,i}(t) = X_i(t) + S_i(t). \tag{8}$$

Considering the self-healing capability of components during the SFP, the degradation increment Y_{ik} will change due to the self-healing when the random shock occurs. Y_{ik} is connected with random shock load W_{ik} and self-healing factor γ, as described by

$$Y_{ik} = c_1 \cdot W_{ik} \cdot e^{-\gamma_i k}. \tag{9}$$

When the number of random shocks is more than 0, the distribution of the degradation increment $S_i(t)$ changes as follows

$$S_i(t) = \sum_{k=1}^{N(t)} Y_{ik} = \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k}. \tag{10}$$

As the self-healing factor γ_i is always greater than 0, $e^{-\gamma ik}$ is less than 1. When γ_i is less than 0, it holds no physical meaning. When γ_i is equal to 0, it indicates that the component lacks self-healing capabilities.

As the random shock load follows a normal distribution, the mean of degradation increments $S_i(t)$ is given as

$$\mu_{S_i(t)} = c_1 \cdot \mu_{w_{ik}} \cdot \sum_{k=0}^{N(t)} e^{-\gamma_i k} = c_1 \cdot \mu_{w_{ik}} \cdot \frac{e^{\gamma_i}}{e^{\gamma_i} - 1}. \tag{11}$$

The variance of the sum of the degradation increments $S_i(t)$ is obtained by

$$\sigma^2_{S_i(t)} = c_1^2 \cdot \sigma^2_{w_{ik}} \cdot \sum_{k=0}^{N(t)} \left(e^{-\gamma_i k}\right)^2 = c_1^2 \cdot \sigma^2_{w_{ik}} \cdot \frac{e^{2\gamma_i}}{e^{2\gamma_i} - 1}. \tag{12}$$

The total degradation $X_{s,i}(t)$ of ith component can be given as follows

$$\begin{aligned} X_{s,i}(t) &= X_i(t) + S_i(t) \\ &= X_i(t) + \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k}. \end{aligned} \tag{13}$$

When there are no shocks, $N(t) = 0$, the probability of no soft failure occurring at time t is derived by

$$\begin{aligned} &P_{i,\mathrm{SF1}}(X_{s,i}(t) H_i | N(t) = 0) \\ &\quad = P(\varphi_i + \beta_{i0} t H_i | N(t) = 0) \\ &\quad = \Phi\left(\frac{H_i - (\varphi_i + \mu_{\beta_{i0}} t)}{\sqrt{\sigma^2_{\beta_{i0}} t^2 + \sigma^2_{\varepsilon_k}}}\right). \end{aligned} \tag{14}$$

When $N(t) = m > 0$, the probability that no soft failure occurs at time t can be obtained as

$$\begin{aligned} P_{i,\mathrm{SF2}}(t) &= P\{X_{s,i}(t) < H_i | N(t) = m\} \\ &= P\Bigg\{\varphi_i + \beta_{i0} t_1 + \beta_{i1} t_2 + \cdots + \beta_{ij} t_{j+1} + \varepsilon_i \\ &\quad + \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k} < H_i | N(t) = m\Bigg\} \\ &= \Bigg(\int_0^t \cdots \Bigg(\int_0^{t_j} P(\varphi_i + \beta_{i0} t_1 + \beta_{i1} t_2 + \cdots + \beta_{ij} t_{j+1} + \varepsilon_i \\ &\quad + \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k} < H_i\Bigg) f_{T_j}(t_j)\,dt_j\Bigg) \cdots f_{T_1}(t_1)\,dt_1\Bigg). \end{aligned} \tag{15}$$

The soft failure reliability of the ith component is derived as

$$
\begin{aligned}
R_{i,\mathrm{SF}} &= R_{i,\mathrm{SF1}} + R_{i,\mathrm{SF2}} \\
&= \Phi\left(\frac{H_i - (\varphi_i + \mu_{\beta_{i0}} t)}{\sqrt{\sigma_{\beta_{i0}}^2 t^2 + \sigma_{\varepsilon_k}^2}}\right) \cdot \mathrm{e}^{-\lambda t} \\
&+ \sum_{m=1}^{\infty} \left\{ \int_0^t \cdots \left[\begin{array}{l} \int_0^{t_j} \Phi\left(\frac{H_i - \left(\varphi_i + \mu_{\beta_{i0}} t_1 + \mu_{\beta_{i1}} t_2 + \cdots + \mu_{\beta_{im}} t_{m+1} + \mu_{S_i(t)}\right)}{\sqrt{\sigma_{\beta_{i0}}^2 t_1^2 + \cdots + \sigma_{\beta_{im}}^2 t_{m+1}^2 + \sigma_{\varepsilon_k}^2 + \sigma_{S_i(t)}^2}}\right) \\ \frac{(\lambda p_2)^{k_m}}{(k_m - 1)!} t_m^{k_m - 1} \mathrm{e}^{-\lambda p_2 t_m} \mathrm{d}t_m \end{array} \right] \cdots \lambda p_2 e^{-\lambda p_2 t_1} \mathrm{d}t_1 \right\} \cdot \frac{\mathrm{e}^{-\lambda t} (\lambda t)^m}{m!}.
\end{aligned} \tag{16}
$$

3.1.1. Reliability model of HFP. It is obvious that the random shock process is related to the degradation process. The specific calculation process of reliability model under different shock models is introduced as follow.

3.1.2. Extreme shock model. The extreme shock model is widely used to describe the random shock. The component will fail when the level of random shock exceeds the HFT.

Assumed that the HFT of ith component changes after each shock. Since the HFT is influenced by the random shock, it is obtained from the load W_{ik} and the transfer parameter c_2. Therefore, we have

$$
\begin{aligned}
D_i &= D_{i0} - \sum_{k=1}^{N(t)} q(Z_i) \\
&= D_{i0} - c_2 \left(\sum_{k=1}^{N(t)} Z_i \right).
\end{aligned} \tag{17}
$$

Considering the self-healing mechanism in HFP, the Z_i will change with the numbers of shock. Z_i is related to the random shock load W_{ik} and self-healing factor ζ_1, as described by

$$
Z_i = W_{ik} \cdot \mathrm{e}^{\zeta_1 (N(t) - k)}, \tag{18}
$$

and the dynamic HFT can be described as

$$
\begin{aligned}
D_i &= D_{i0} - \sum_{k=1}^{N(t)} q(Z_i) \\
&= D_{i0} - c_2 \left(\sum_{k=1}^{N(t)} W_{ik} \cdot \mathrm{e}^{\zeta_1 (N(t) - k)} \right).
\end{aligned} \tag{19}
$$

Thus, the probability that the component avoids failure in HFP at time t corresponds to the random shock load remaining below its dynamic HFT. This probability is described by the following equation (20)

$$
\begin{aligned}
&P(W_{ik} < D_i) \\
&= P\left(W_{ik} < D_{i0} - c_2 \left(\sum_{k=1}^{N(t)} W_{ik} \cdot \mathrm{e}^{\zeta_1 (N(t)-k)} \right) \right) \\
&= \sum_{m=1}^{\infty} P\left(W_{ik} + \left(c_2 \sum_{k=1}^{N(t)} W_{ik} \cdot \mathrm{e}^{\zeta_1 (N(t)-k)} \right) D_{i0} | N(t) = m \right) \\
&= \sum_{m=1}^{\infty} P(G_1 < D_{i0}).
\end{aligned} \tag{20}
$$

Moreover, according to mathematical derivation:

$$
\begin{aligned}
\mu_1 &= \mu_{W_{ik}} \left(\mathrm{e}^{\zeta_1 (N(t)-1)} + \mathrm{e}^{\zeta_1 (N(t)-2)} + \cdots + 1 \right) \\
&= \sum_{k=1}^{N(t)} \mu_{W_{ik}} \mathrm{e}^{\zeta_1 (N(t)-k)} = \frac{1 - \mathrm{e}^{\zeta_1 N(t)}}{1 - \mathrm{e}^{\zeta_1}} \mu_{W_{ik}}, \\
\sigma_1^2 &= \sigma_{W_{ik}}^2 \left(\mathrm{e}^{2\zeta_1 (N(t)-1)} + \mathrm{e}^{2\zeta_1 (N(t)-2)} + \cdots + 1 \right) \\
&= \sum_{k=1}^{N(t)} \sigma_{W_{ik}}^2 \mathrm{e}^{2\zeta_1 (N(t)-k)} = \frac{1 - \mathrm{e}^{2\zeta_1 N(t)}}{1 - \mathrm{e}^{2\zeta_1}} \sigma_{W_{ik}}^2.
\end{aligned} \tag{21}
$$

Therefore, the mean and variance of G_1 can be expressed as

$$
\begin{aligned}
G_1 \sim N\Bigg(& \mu_{W_{ik}} + c_2 \left(\frac{1 - \mathrm{e}^{\zeta_1 m}}{1 - \mathrm{e}^{\zeta_1}} \mu_{W_{ik}} \right), \sigma_{W_{ik}}^2 \\
& + c_2^2 \left(\frac{1 - \mathrm{e}^{2\zeta_1 m}}{1 - \mathrm{e}^{2\zeta_1}} \sigma_{W_{ik}}^2 \right) \Bigg).
\end{aligned} \tag{22}
$$

Based on the equations (20) and (22), the probability of the component avoiding hard failure at time t is derived by

$$
\begin{aligned}
P_{i,HF1}(t) &= P\left(\bigcap_{k=0}^{m} W_{ik}(T_k) D_i | N(t) = m \right) = F_W(D_i)^m \\
&= \Phi\left(\frac{D_{i0} - \mu_{W_{ik}} + c_2 \left(\frac{1 - \mathrm{e}^{\zeta_1 m}}{1 - \mathrm{e}^{\zeta_1}} \mu_{W_{ik}} \right)}{\sqrt{\sigma_{W_{ik}}^2 + c_2^2 \left(\frac{1 - \mathrm{e}^{2\zeta_1 m}}{1 - \mathrm{e}^{2\zeta_1}} \sigma_{W_{ik}}^2 \right)}} \right)^m.
\end{aligned} \tag{23}
$$

3.1.3. Cumulative shock model. Hard failure occurs when the cumulative shock load $\sum W_{ik}$ exceeds the hard failure threshold D_i of the ith component. The total shock load $W_i(t)$ at time t is determined with random shock load and self-healing factor ζ_2, which is expressed as

$$W_i(t) = \sum_{k=1}^{N(t)} c_3 \cdot W_{ik} \cdot \mathrm{e}^{-\zeta_2(N(t)-k)}. \quad (24)$$

Therefore, the probability that the sum of the random shock loads $\sum W_{ik}$ does not exceed the HFT D_i is given as

$$\begin{aligned} P(W_i(t) < D_i) &= P\left(\sum_{k=1}^{N(t)} c_3 \cdot W_{ik} \cdot \mathrm{e}^{-\zeta_2(N(t)-k)} < D_{i0}\right) \\ &= \sum_{m=0}^{\infty} P\left(\sum_{k=1}^{N(t)} c_3 \cdot W_{ik} \cdot \mathrm{e}^{-\zeta_2(N(t)-k)}\ D_{i0} | N(t) = m\right) \\ &= \sum_{m=0}^{\infty} P(G_2 < D_{i0}). \end{aligned} \quad (25)$$

Similar to G_1, the mean and variance of G_2 can be expressed as

$$G_2 \sim N\left(c_3 \cdot \frac{1-\mathrm{e}^{-\zeta_2 N(t)}}{1-\mathrm{e}^{-\zeta_2}} \mu_{W_{ik}}, c_3^2 \cdot \frac{1-\mathrm{e}^{-2\zeta_2 N(t)}}{1-\mathrm{e}^{-2\zeta_2}} \sigma_{W_{ik}}^2\right). \quad (26)$$

Thus, the probability of the component avoiding hard failure at time t is derived by

$$\begin{aligned} P_{i,\mathrm{HF2}}(t) &= P\left(\sum_{m=0}^{N(t)} W(T_m) D(t) | N(t) = m\right) \\ &= \Phi\left(\frac{D_{i0} - c_3 \cdot \frac{1-\mathrm{e}^{-\zeta_2 m}}{1-\mathrm{e}^{-\zeta_2}} \cdot \mu_{W_{ik}}}{\sqrt{c_3^2 \cdot \frac{1-\mathrm{e}^{-2\zeta_2 m}}{1-\mathrm{e}^{-2\zeta_2}} \cdot \sigma_{W_{ik}}^2}}\right)^m. \end{aligned} \quad (27)$$

3.2. Reliability model based on DCFP

The failure of each component is based on DCFP. Therefore, the component will fail, whether soft failure or hard failure occurs. As a result, the reliability can be obtained by calculating the probability that neither soft failure nor hard failure occurs.

3.2.1. Extreme shock model. Based on extreme shock model, the reliability expression can be given by

$$R_{i,e}(t) = P\{X_{s,i}(t) < H_i\} P\{W_{ik} < D_i(t)\}. \quad (28)$$

To calculate the reliability more conveniently, two cases are considered in this study: the number of shocks is 0 or not, *i.e.,* $N(t) = 0$ and $N(t) = m > 0$.

(1) When no random shock occurs at time *t, i.e.,* $N(t) = 0$, the reliability is only related to the degradation process. In this case, the reliability can be expressed as

$$R_{i,e1} = R_{i,\mathrm{SF1}} = \Phi\left(\frac{H_i - (\varphi_i + \mu_{\beta_{i0}} t)}{\sqrt{\sigma_{\beta_{i0}}^2 t^2 + \sigma_{\varepsilon_k}^2}}\right) \cdot \mathrm{e}^{-\lambda t}. \quad (29)$$

(2) When the number of random shocks at time t is greater than 0, *i.e.,* $N(t) = m$, where $m > 0$, the reliability function is given by

$$\begin{aligned} R_{i,e2}(t | N(t) = m) = \sum_{m=1}^{\infty} P\left(X_{s,i}(t) < H_i, \bigcap_{k=0}^{N(t)} W_{ik} < D_i(t) | N(t) = m\right) \\ \cdot P(N(t) = m). \end{aligned} \quad (30)$$

Based on the previous discussion, when the number of random shocks at time t is greater than 0, the reliability expression for the ith component can be derived as

$$
\begin{aligned}
R_{i,e2}(t) &= (t|N(t)=m) \\
&= \sum_{m=1}^{\infty} P\left(X_{s,i}(t) < H_i, \bigcap_{k=0}^{m} P\left(W_{ik} < D_i(t)\right)\right) \cdot P(N(t)=m) \\
&= \sum_{m_1=1}^{\infty} \left[\begin{array}{l} \left(\int_0^t \cdots \left(\int_0^{t_j} P \left(\begin{array}{l} \varphi_i + \beta_{i0} t_1 + \beta_{i1} t_2 + \cdots + \beta_{ij} t_{j+1} \\ + \varepsilon_i + \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k} < H_i \end{array} \right) f_{T_j}(t_j)\, dt_j \right) \cdots f_{T_1}(t_1)\, dt_1 \right) \\ \times \Phi \left(\dfrac{D_{i0} - \mu_{W_{ik}} + c_2 \left(\frac{1-e^{\zeta_1 m}}{1-e^{\zeta_1}} \mu_{W_{ik}} \right)}{\sqrt{\sigma_{W_{ik}}^2 + c_2^2 \left(\frac{1-e^{2\zeta_1 m}}{1-e^{2\zeta_1}} \sigma_{W_{ik}}^2 \right)}} \right)^m \times P(N(t)=m) \end{array} \right] \\
&= \sum_{m_1=1}^{\infty} \left[\begin{array}{l} \left(\int_0^t \cdots \left(\int_0^{t_j} P \left(\begin{array}{l} \varphi_i + \beta_{i0} t_1 + \beta_{i1} t_2 + \cdots + \beta_{ij} t_{j+1} \\ + \varepsilon_i + \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k} < H_i \end{array} \right) f_{T_j}(t_j)\, dt_j \right) \cdots f_{T_1}(t_1)\, dt_1 \right) \\ \times \Phi \left(\dfrac{D_{i0} - \mu_{W_{ik}} + c_2 \left(\frac{1-e^{\zeta_1 m}}{1-e^{\zeta_1}} \mu_{W_{ik}} \right)}{\sqrt{\sigma_{W_{ik}}^2 + c_2^2 \left(\frac{1-e^{2\zeta_1 m}}{1-e^{2\zeta_1}} \sigma_{W_{ik}}^2 \right)}} \right)^m \times \left(\dfrac{e^{(-\lambda t)} \cdot (\lambda t)^m}{(m)!} \right) \end{array} \right].
\end{aligned} \tag{31}
$$

The reliability of the ith component under extreme shock model is

$$
\begin{aligned}
& R_{i,e}(t|N(t)=m) = R_{i,e1}(t) + R_{i,e2}(t) \\
& = \Phi \left(\frac{H_i - (\varphi_i + \mu_{\beta_{i0}} t)}{\sqrt{\sigma_{\beta_{io}}^2 t^2 + \sigma_{\varepsilon_k}^2}} \right) \cdot e^{-\lambda t} + \sum_{m=1}^{\infty} \left[\begin{array}{l} \left(\int_0^t \cdots \left(\int_0^{t_j} P \left(\begin{array}{l} \varphi_i + \beta_{i0} t_1 + \beta_{i1} t_2 + \cdots + \beta_{ij} t_{j+1} \\ + \varepsilon_i + \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k} < H_i \end{array} \right) f_{T_j}(t_j)\, dt_j \right) \cdots f_{T_1}(t_1)\, dt_1 \right) \\ \times \Phi \left(\dfrac{D_{i0} - \mu_{W_{ik}} + c_2 \left(\frac{1-e^{\zeta_1 m}}{1-e^{\zeta_1}} \mu_{W_{ik}} \right)}{\sqrt{\sigma_{W_{ik}}^2 + c_2^2 \left(\frac{1-e^{2\zeta_1 m}}{1-e^{2\zeta_1}} \sigma_{W_{ik}}^2 \right)}} \right)^m \times \left(\dfrac{e^{(-\lambda t)} \cdot (\lambda t)^m}{(m)!} \right) \end{array} \right].
\end{aligned} \tag{32}
$$

3.2.2. Cumulative shock model. Similar to extreme shock model, the two cases under cumulative shock model are also considered: $N(t) = 0$ and $N(t) = m > 0$.

(1) When no random shock occurs at time t, *i.e.,* $N(t) = 0$, the reliability expression is the same as $R_{i,e1}(t)$

$$
R_{i,c1}(t|N(t)=0) = \Phi \left(\frac{H_i - (\varphi_i + \mu_{\beta_{i0}} t)}{\sqrt{\sigma_{\beta_{i0}}^2 t^2 + \sigma_{\varepsilon_k}^2}} \right) \cdot \mathrm{e}^{-\lambda t}. \tag{33}
$$

(2) When the number of random shocks at time t is more than 0, *i.e.,* $N(t) = m > 0$

$$
\begin{aligned}
R_{i,c2}(t) &= (t|N(t)=m) \\
&= \sum_{m=1}^{\infty} P\left(X_{s,i}(t) < H_i, W_i(t) < D_i | N(t)=m\right) \times P(N(t)=m) \\
&= \sum_{m=1}^{\infty} \left[\begin{array}{l} \left(\int_0^t \cdots \left(\int_0^{t_j} P \begin{pmatrix} \varphi_i + \beta_{i0} t_1 + \beta_{i1} t_2 + \cdots + \beta_{ij} t_{j+1} \\ +\varepsilon_i + \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k} < H_i \end{pmatrix} f_{T_j}(t_j) \, dt_j \right) \cdots f_{T_1}(t_1) \, dt_1 \right) \\ \times \Phi \left(\dfrac{D_{i0} - c_3 \cdot \frac{1-e^{-\zeta_2 m}}{1-e^{-\zeta_2}} \cdot \mu_{W_{ik}}}{\sqrt{c_3^2 \cdot \frac{1-e^{-2\zeta_2 m}}{1-e^{-2\zeta_2}} \cdot \sigma_{W_{ik}}^2}} \right)^m \times P(N(t)=m) \end{array} \right] . \qquad (34) \\
&= \sum_{m_1=1}^{\infty} \left[\begin{array}{l} \left(\int_0^t \cdots \left(\int_0^{t_j} P \begin{pmatrix} \varphi_i + \beta_{i0} t_1 + \beta_{i1} t_2 + \cdots + \beta_{ij} t_{j+1} \\ +\varepsilon_i + \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k} < H_i \end{pmatrix} f_{T_j}(t_j) \, dt_j \right) \cdots f_{T_1}(t_1) \, dt_1 \right) \\ \times \Phi \left(\dfrac{D_{i0} - c_3 \cdot \frac{1-e^{-\zeta_2 m}}{1-e^{-\zeta_2}} \cdot \mu_{W_{ik}}}{\sqrt{c_3^2 \cdot \frac{1-e^{-2\zeta_2 m}}{1-e^{-2\zeta_2}} \cdot \sigma_{W_{ik}}^2}} \right)^m \times \left(\dfrac{e^{(-\lambda t)} \cdot (\lambda t)^m}{(m)!} \right) \end{array} \right]
\end{aligned}
$$

Therefore, the general reliability expression for the ith component can be derived as

$$
\begin{aligned}
R_{i,c}(t|N(t)=m) &= R_{i,c1}(t) + R_{i,c2}(t) \\
&= \Phi \left(\frac{H_i - (\varphi_i + \mu_{\beta_{i0}} t)}{\sqrt{\sigma_{\beta_{io}}^2 t^2 + \sigma_{\varepsilon_k}^2}} \right) \cdot e^{-\lambda t} \\
&+ \sum_{m=1}^{\infty} \left[\begin{array}{l} \left(\int_0^t \cdots \left(\int_0^{t_j} P \begin{pmatrix} \varphi_i + \beta_{i0} t_1 + \beta_{i1} t_2 + \cdots + \beta_{ij} t_{j+1} \\ +\varepsilon_i + \sum_{k=1}^{N(t)} c_1 \cdot W_{ik} \cdot e^{-\gamma_i k} < H_i \end{pmatrix} f_{T_j}(t_j) \, dt_j \right) \cdots f_{T_1}(t_1) \, dt_1 \right) \times \Phi \left(\dfrac{D_{i0} - c_3 \cdot \frac{1-e^{-\zeta_2 m}}{1-e^{-\zeta_2}} \cdot \mu_{W_{ik}}}{\sqrt{c_3^2 \cdot \frac{1-e^{-2\zeta_2 m}}{1-e^{-2\zeta_2}} \cdot \sigma_{W_{ik}}^2}} \right)^m \\ \times \left(\dfrac{e^{(-\lambda t)} \cdot (\lambda t)^m}{(m)!} \right) \end{array} \right] . \qquad (35)
\end{aligned}
$$

3.2.3. Reliability modeling of multi-component systems. A multi-component system is composed of components exhibiting varying performances. Each component of the system undergoes two interdependent and competing failure processes. The failure time of the system is influenced by the behavior of all components. The reliability of multi-component system can be regarded as the probability of the intersection of multiple events. The failure of system is defined as the condition in which the total degradation does not exceed the soft failure threshold and the components exposed to external shocks do not fail catastrophically [14]. The system composed of $k_1 + k_2 + k_3 + \cdots + k_n$ components is illustrated in figure 3.

The general expression for the parallel-series system reliability expression is given as

$$
\begin{aligned}
R_s(t) &= \sum_{n=0}^{\infty} R(t|N(t)=n) \times P\{N(t)=n\} \\
&= \sum_{n=0}^{\infty} \left\{ 1 - \prod_{p=1}^{n} \left[1 - \prod_{q=1}^{k_p} R_{pq}(t|N(t)=n) \right] \right\} \\
&\quad \times P\{N(t)=n\} \\
&= \sum_{n=0}^{\infty} \left\{ 1 - \prod_{p=1}^{n} \left[1 - \prod_{q=1}^{k_p} p_1 p_2 \right] \right\} P\{N(t)=n\} . \qquad (36)
\end{aligned}
$$

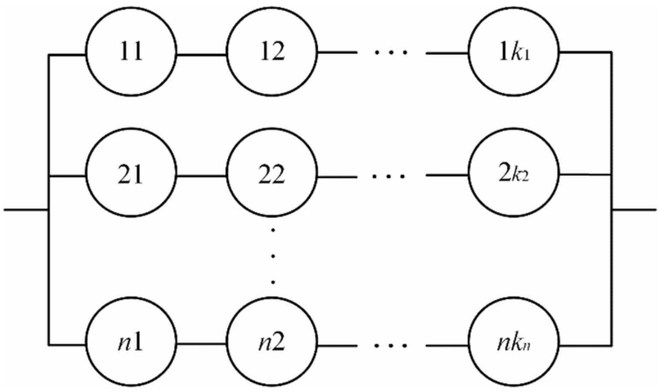


Figure 3. A general parallel-series system with self-healing.

where $p_1 = P\{W_{pq}(t) \leqslant D_{pq}\}$, $p_2 = P\{X_{s,pq}(t) \leqslant H_{pq}\}$.

Based on the above equation, the system reliability expression under the extreme shock model is derived as

$$
\begin{aligned}
R_{s,e}(t) &= \sum_{n=0}^{\infty}\left\{1-\prod_{p=1}^{n}\left[1-\prod_{q=1}^{k_p} p_1 p_2\right]\right\} \times \left(\frac{e^{(-\lambda t)}\cdot(\lambda t)^n}{(n)!}\right) \\
&= \sum_{n=0}^{\infty}\left\{1-\prod_{p=1}^{n}\left[1-\prod_{q=1}^{k_p}\left[\begin{array}{l}\Phi\left(\dfrac{D_{pq}-\mu_{W_{pq}}+c_2\left(\frac{1-e^{\zeta_1 n}}{1-e^{\zeta_1}}\mu_{W_{pq}}\right)}{\sqrt{\sigma^2_{W_{pq}}+c_2^2\left(\frac{1-e^{2\zeta_1 n}}{1-e^{2\zeta_1}}\sigma^2_{W_{pq}}\right)}}\right) \\ \times\Phi\left(\dfrac{H_{pq}-\left(\varphi_{pq}+\mu_{\beta_{pq0}}t_1+\mu_{\beta_{pq1}}t_2+\cdots+\mu_{\beta_{pqm}}t_{m+1}+\mu_{S_{pq}(t)}\right)}{\sqrt{\sigma^2_{\beta_{pq0}}t_1^2+\cdots+\sigma^2_{\beta_{pqm}}t_{m+1}^2+\sigma^2_{\varepsilon_k}+\sigma^2_{S_{pq}(t)}}}\right)\end{array}\right]\right]\right\} \times \left(\frac{e^{(-\lambda t)}\cdot(\lambda t)^n}{(n)!}\right).
\end{aligned} \tag{37}
$$

The system reliability expression under the cumulative shock model can be derived as

$$
\begin{aligned}
R_{s,c}(t) &= \sum_{n=0}^{\infty}\left\{1-\prod_{p=1}^{n}\left[1-\prod_{q=1}^{k_p} p_1 p_2\right]\right\} \times \left(\frac{e^{(-\lambda t)}\cdot(\lambda t)^n}{(n)!}\right) \\
&= \sum_{n=0}^{\infty}\left\{1-\prod_{p=1}^{n}\left[1-\prod_{q=1}^{k_p}\left[\begin{array}{l}\Phi\left(\dfrac{D_{pq}-\frac{1-e^{-\zeta_2 n}}{1-e^{-\zeta_2}}\mu_{W_{pq}}}{\sqrt{\frac{1-e^{-2\zeta_2 n}}{1-e^{-2\zeta_2}}\sigma^2_{W_{pq}}}}\right) \\ \times\Phi\left(\dfrac{H_{pq}-\left(\varphi_{pq}+\mu_{\beta_{pq0}}t_1+\mu_{\beta_{pq1}}t_2+\cdots+\mu_{\beta_{pqm}}t_{m+1}+\mu_{S_{pq}(t)}\right)}{\sqrt{\sigma^2_{\beta_{pq0}}t_1^2+\cdots+\sigma^2_{\beta_{pqm}}t_{m+1}^2+\sigma^2_{\varepsilon_k}+\sigma^2_{S_{pq}(t)}}}\right)\end{array}\right]\right]\right\} \times \left(\frac{e^{(-\lambda t)}\cdot(\lambda t)^n}{(n)!}\right).
\end{aligned} \tag{38}
$$

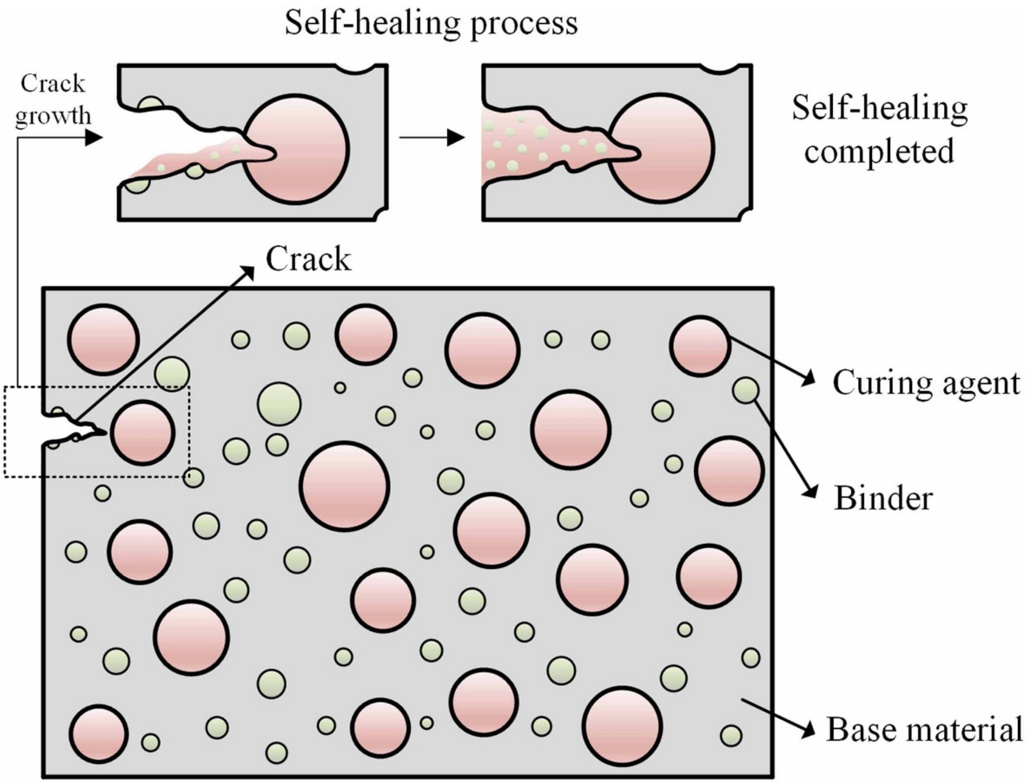


Figure 4. Self-healing process demonstration.

4. Numerical examples

The gearbox is the most widely used component of transmission for changing rotational speed and transmitting power, which fails due to wear, cracks, and imbalance mostly. Wear is one of the most common types of failures in gearbox. The sliding between two gears causes wear on the contact surfaces of the components. Additionally, the impurities infiltrated in the gearbox or the electrochemical reactions both can lead to surface damage of the components. There are many kinds of friction self-repair mechanisms, including adsorption self-repair mechanism, tribo-chemical self-repair mechanism, penetration self-repair mechanism, glazing self-repair mechanism, nano-metal particle self-repair mechanism, and self-repair mechanism using micro-nano mineral powder additives. The micro-damage of materials can be healed automatically through the mechanism of material or Energy supply mechanism.

The self-healing gearbox considered in this case study achieved degradation recovery and impact resistance recovery through self-healing materials. The self-healing of bearings and gears in gearbox mainly involves the addition of oil-soluble anti-friction additives containing polar elements or non-oil-soluble solid particles. During the operation of gearbox, these substances will diffuse to the microscopic friction surface, where undergo tribo-chemical reactions with the worn surfaces of the gears or bearings. This process results in the deposition of material that fills the uneven worn areas, forming a repair film with anti-wear and friction-reducing properties. The repair film improves the lubrication performance of the friction surfaces, which prevents direct contact between the interacting components, reduces friction and wear, and ultimately extends the service life of the gearbox.

In addition, external shocks can also cause damage to self-healing gearbox. For the microcracks in gears and bearings, external self-healing materials can be employed to make self-healing of the crack. The external self-healing technique involves placing fiber conduits or microcapsules in areas that prone to cracking in the gearbox components. The fiber ducts or microcapsules at the crack which are in front of the gear or bearing are broken when the matrix material cracks under external actions. The internal adhesive flows out and seeps into the cracks under capillary action, mixing with the curing agent. A curing reaction bonds the crack surfaces together, which prevents the crack propagation furtherly and achieves the effect of self-healing. The self-healing process is illustrated in figure 4.

This section conducts simulation analysis and validation of the reliability model developed in section 3 by using a self-healing gearbox case study.

The multi-component gearbox system developed in this section is shown in figure 5, consisting of four gears, which exist self-healing capabilities for both SFP and HFP. Each gear is subjected to two interdependent and competing failure processes. The soft failure is caused by internal degradation of the gear, while the hard failure is induced by external mechanical vibrations. The random shock affects all gear components within the gearbox system, while the damage caused by external shocks accumulates on each component.

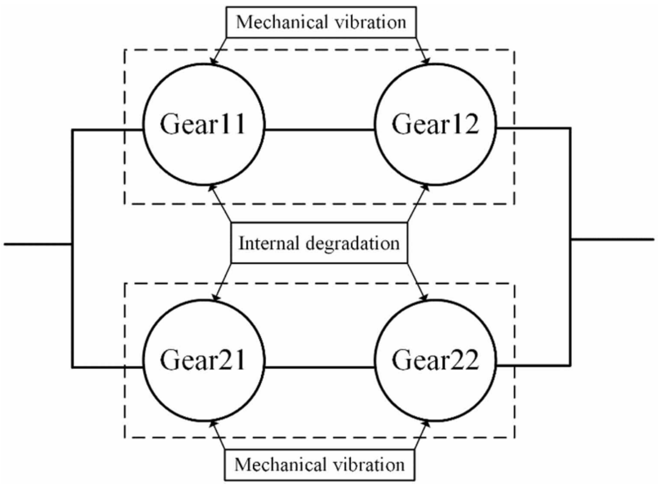


Figure 5. The gearbox system consisting of four gears with self-healing.

Table 1. Parameter values.

Parameters	Values	Sources
Di	1.4283 GPa	Vivek Srivastava *et al* [38]
Hi	0.025 cm	Yi *et al* [13]
$\mu\beta i$	8.4823 × 10−7 cm	Lyu *et al* [39]
$\sigma\beta i$	6.0016 × 10−8 cm	Lyu *et al* [39]
φi	0 cm	Assumption
λ	0.1 h^{-1}	Lyu *et al* [39]
μWi	0.4 GPa	Lyu *et al* [39]
σWi	0.02 GPa	Lyu *et al* [39]
$c1$	0.85	Assumption
$c2$	0.80	Assumption
$c3$	0.60	Assumption
γ	2.5×10^{-2}	Assumption
$\zeta1, \zeta2$	0.6	Assumption

The basic parameters of each gear component i for $i = 1,2,3,4$ are provided in table 1. Since each gear component has self-healing capabilities, the self-recovery can be adjusted by modifying the self-healing factors for both the SFP and the HFP.

4.1. System reliability analysis

Based on the data listed in table 1 and the function in equations (37) and (38), the system reliability curves under two different shock models are shown in figure 6. The comparison of the two curves indicates that the system reliability under cumulative shock model is better than that under extreme shock model. The result indicates that self-healing materials perform better for cumulative damage than for extreme damage.

The four curves in figure 7 compare the reliability of system model presented in this study with other three types of systems including system with self-healing of SFP, system with self-healing of HFP, and system without self-healing. The results indicate that, in comparison to the normal system, the reliability curve of the system shifts to the right when it possesses self-healing capabilities. The most noticeable rightward shift occurs when the system exists self-healing capabilities in SFP and HFP simultaneously, which indicates that the system reliability is significantly higher than that of the other three systems. In fact, the self-healing factors γ improves the capability of gear to resist internal degradation, reducing the degradation increment induced by random shocks, which enhances soft failure reliability of gears. The application of self-healing factors ζ_1 and ζ_2 improves the ability of gears to avoid hard failure under random shocks, which enhances its hard failure reliability. As a result, the overall reliability of the gear system is improved.

4.2. Sensitivity analysis

The sensitivity analysis of the self-healing factor γ for SFP under two different shock models is presented in figures 8 and 9. It can be observed that as the self-healing factor γ increases exponentially from 2.5×10^{-3}–2.5×10^{-1}, the reliability curve shifts to the right slowly. With the increase in γ, the internal self-healing ability of gears becomes stronger, and the system reliability gradually improves. However, the rightward shift of the system reliability curve is not significant, which indicates that the increase in γ has a relatively modest

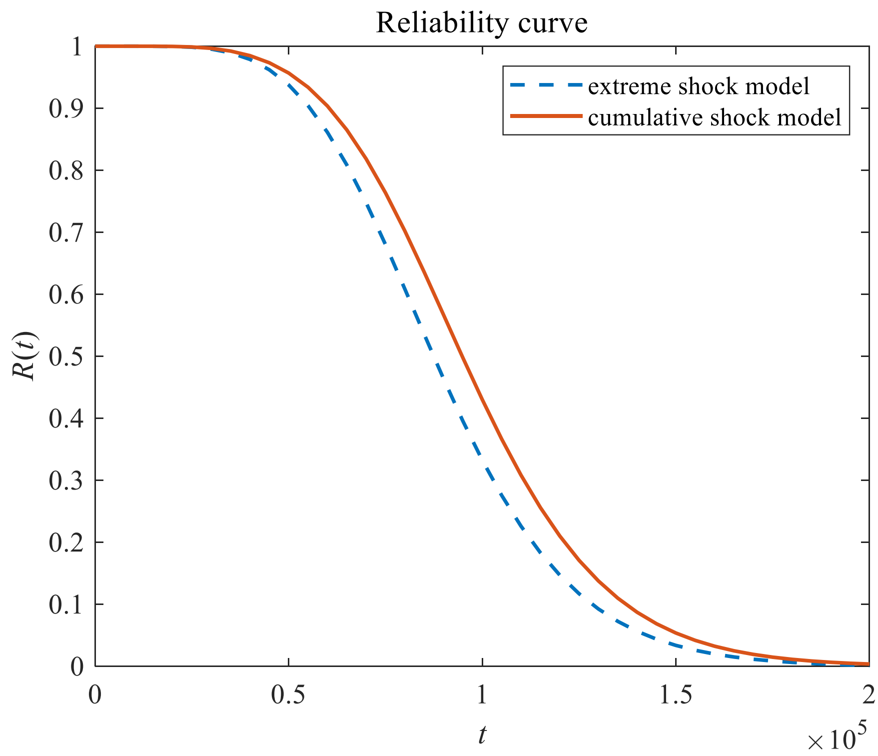


Figure 6. Curves of total reliability under two different shock models.

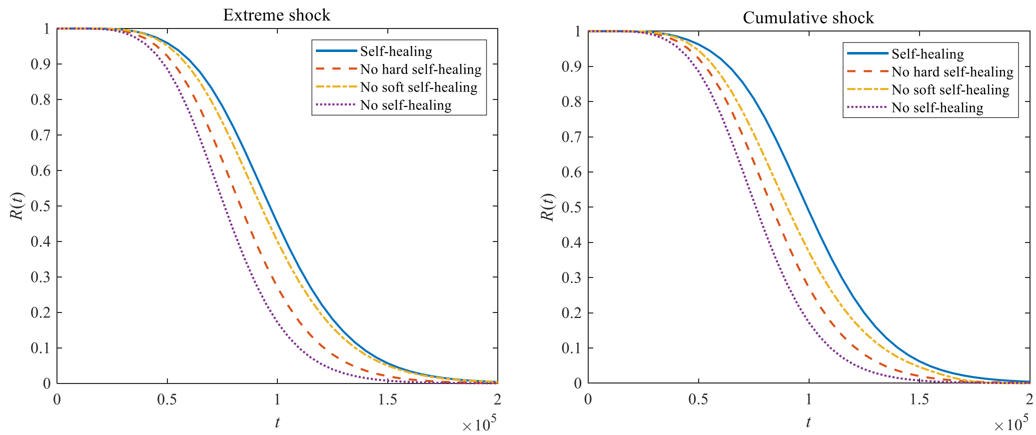


Figure 7. Curves of total reliability between different types of system with self-healing.

influence on system reliability. This is consistent with the practical situation of self-healing gears, as the internal degradation of the gears triggers fewer self-healing materials. The defects caused by internal degradation, such as cracks, have a low probability of occurrence. The internal self-healing material has not been catalyzed or it has not entered the crack, which leads to a less significant effect of self-healing in SFP.

Figure 10 shows the sensitivity analysis of self-healing factor ζ for HFP. In extreme shock model, the ζ affects the recovery level of HFT. In cumulative shock model, the ζ influences the recovery degree of cumulative damage. The reliability curve indicates that as the self-healing factor ζ increases from 0.2 to 0.8, the system reliability curve shifts significantly to the right. The reason is that, with the increase in ζ, the triggering of internal self-healing materials becomes more frequent when the gear generates the cracks caused by random shocks, which results a stronger recovery ability for HFP. Therefore, the reliability of system improves progressively.

The sensitivity analysis for soft failure threshold of the gear is depicted in figure 11. The system reliability curve decreases as the soft failure threshold is reduced. As the initial value of the soft failure threshold H changes from 0.002 cm to 0.0035 cm, the curve progressively shifts to the right. It enables the system to maintain the higher reliability easily. The result indicates that when the soft failure threshold is higher, the

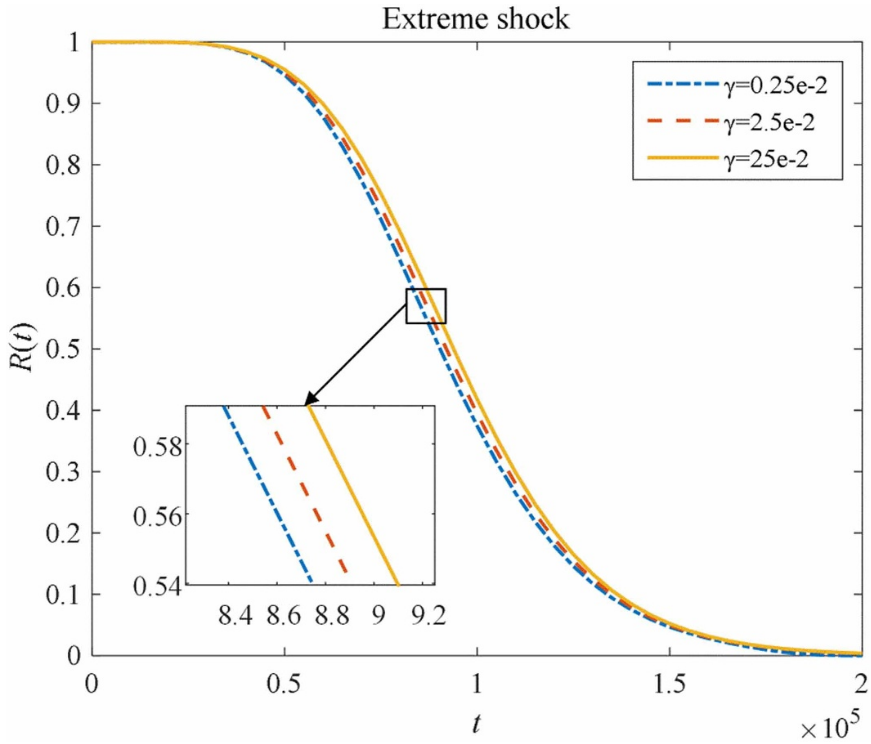


Figure 8. Sensitivity analysis of γ under extreme shock model.

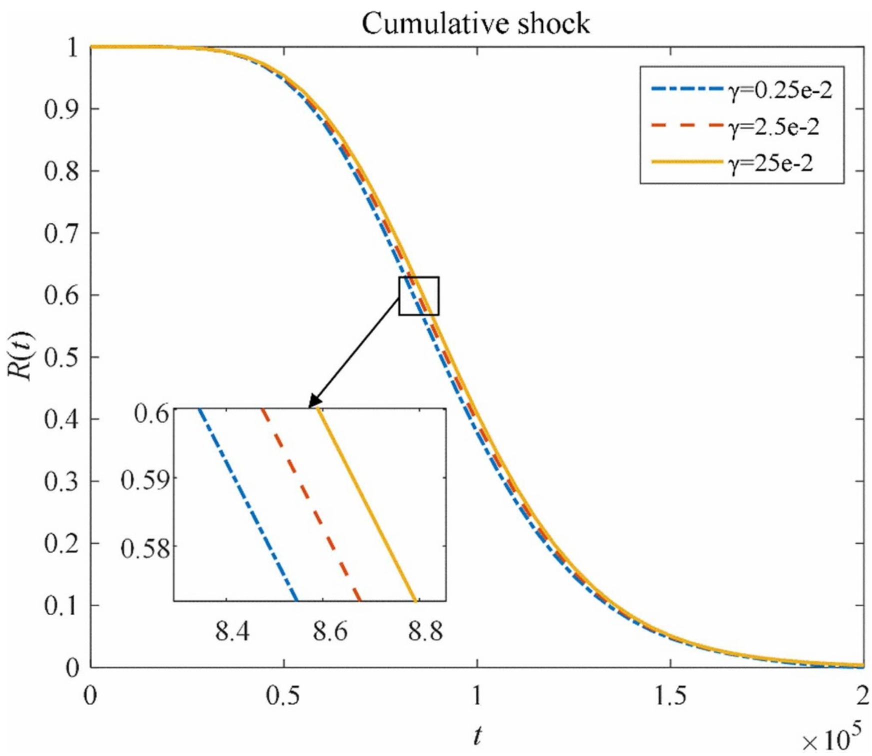


Figure 9. Sensitivity analysis of γ under cumulative shock model.

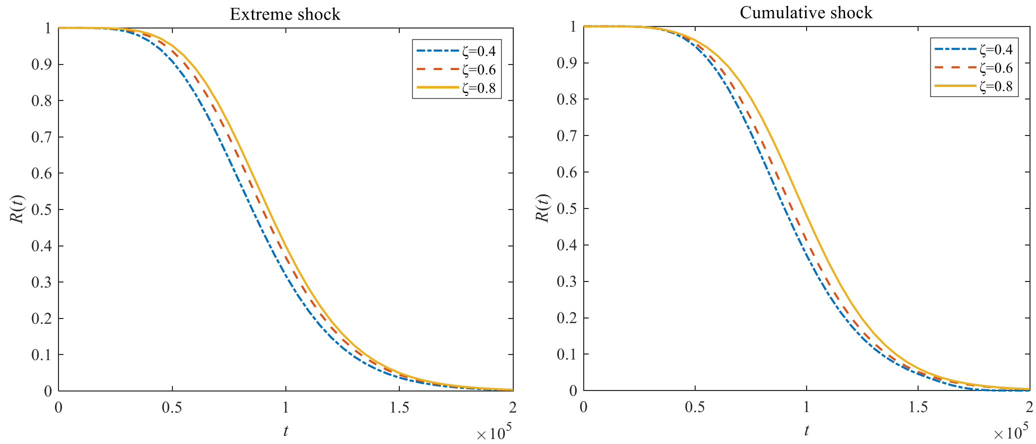


Figure 10. Sensitivity analysis of ζ under two different shock models.

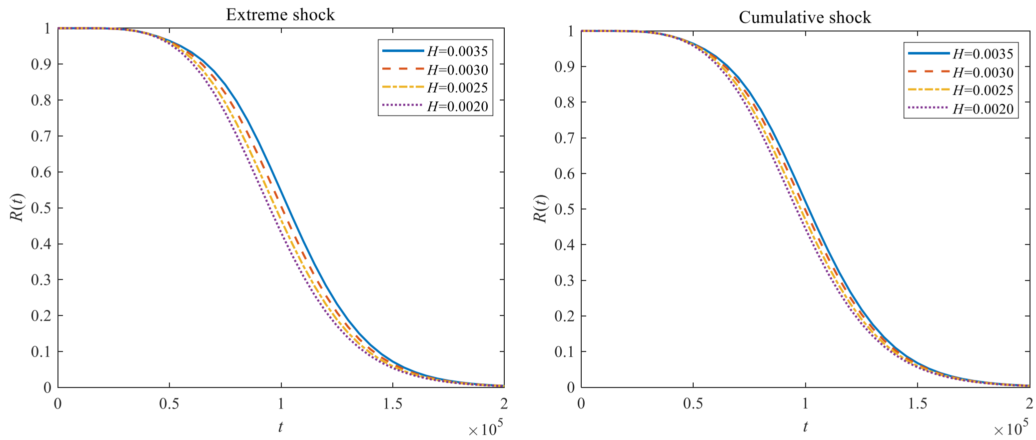


Figure 11. Sensitivity analysis of H under two different shock models.

capability of system to resist to soft failure become stronger, which makes the system more reliable.

Although the validation in this study may be mainly numerical, the predicted reliability trend is consistent with the experimental observations of existing self-healing polymers and composite materials. In these materials, partial recovery of mechanical strength or shock resistance after damage has been widely observed. These existing experimental studies confirm the rationality and reliability of the proposed model, indicating that the reliability assessment results predicted by the model are similar with the self-healing behaviors in practice.

5. Conclusions

In this paper, we study the reliability of the multi-component system with self-healing mechanism based on the DCFP. A reliability model that takes both soft failure self-healing and hard failure self-healing into account is developed to better describe the effect of self-healing on multi-component systems. We investigate the impact of parameters on reliability of system including the failure threshold and self-healing factor.

Self-healing mechanism may mitigate the degradation process or enhance the resilience against catastrophic failures. For the former, the impact of self-healing manifests as the restoration of incremental degradation. For the latter, the influence of self-healing considered in this paper is primarily reflected in two aspects: the restoration of HFT and accumulated damage. Two categories of self-healing factors are introduced to reflect the influence of self-healing mechanisms on both SFP and HFP. The relationship between self-healing factors with the number of random shocks and time of random shocks is derived through exponential functions, and the detailed self-recovery model is established. The first category of self-healing factor reflects the recovery degree of the polymer material to the internal degradation process of system correlated with the degradation increment, while the second-type self-healing factor reflects the capability of the polymer material to recover the system under external shocks. By incorporating the above effects, two types of self-healing models are established through exponential formulas and the reliability of system is analyzed by the full probability formulation and the composite integral method. Finally, a parallel-series system is developed, and a general multi-component reliability model is formulated by considering the competing risks between degradation processes and fatal impacts. The general analytical expression of the model is derived at the same time.

A case study of the gearbox is provided to demonstrate the application of the developed model The results indicate that the random shock and deterioration affect the reliability of each individual component, which in turn influence the overall reliability of the system. Compared with the models in previous studies, the model established in this paper has higher reliability, which can better fit the actual situation. The method has good practicality. The developed reliability model is not only the extension of existing models, but also provides a scheme to analyze reliability of other multi-component systems with-recovery mechanisms. In future work, more research is needed on systems with more complex self-healing situations, such as the effect of partial self-healing or continuous self-healing on system reliability. In addition, the detailed analysis of self-healing behavior is needed to describe the self-healing modes more accurately when subjected to other shock models. Meanwhile, nonlinear degradation processes should be considered to describe the internal degradation process of the system such as the Gamma process. Therefore, future work will focus on developing a more comprehensive reliability assessment method for self-healing system based on this research.

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